

Lecture 37**Module IV - Interlaminar stresses****Inter-laminar Stresses**

A state of stress at a point is normally expressed by the six stress components; namely, σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} , and τ_{yz} . As far as a laminate is concerned, σ_x , σ_y , and τ_{xy} are called as in-plane (normal and shear) stresses and σ_z , τ_{xz} , and τ_{yz} are out-of-plane stresses or inter-laminar (normal and shear) stresses. In a fiber reinforced laminate, the load is transferred to adjacent layers by means of these inter-laminar stresses.

For better understanding, consider a balanced symmetric laminate of $[\pm 45^\circ]_s$ subjected to a uniaxial tensile load N_x ($N_y = N_{xy} = 0$). From CLT, the terms of A and B matrices which are zero are:

$$\begin{aligned} A_{16} = A_{26} &= 0 && \text{because of balanced laminate} \\ [B_{ij}] &= 0 && \text{due to symmetry} \end{aligned}$$

From CLT,

$$[N] = [A] \{ \epsilon^o \} + [B] \{ k \} = [A] \{ \epsilon^o \} \quad (5.1)$$

Therefore the mid-plane strains are,

$$\epsilon_{x^o} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} N_x \quad (5.2)$$

$$\epsilon_{y^o} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2} N_x \quad (5.3)$$

$$\gamma_{xy} = 0 \quad (5.4)$$

The state of stress in any layer may be expressed as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} \overline{Q_{ij}} \end{pmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ 0 \end{Bmatrix}$$

From this relation, it is clear that even though the shear stress resultant N_{xy} is zero, each layer experiences an in-plane shear stress τ_{xy} . Since, there is no applied shear stress, τ_{xy} should be zero at the free edge. i.e. τ_{xy} will have some value in the interior of the laminate and zero at the

boundary. The transition of τ_{xy} value from a finite to zero near the free edge is due to the development of inter-laminar shear stress τ_{xz} near the free edge. This is shown in the following figure 5.1.

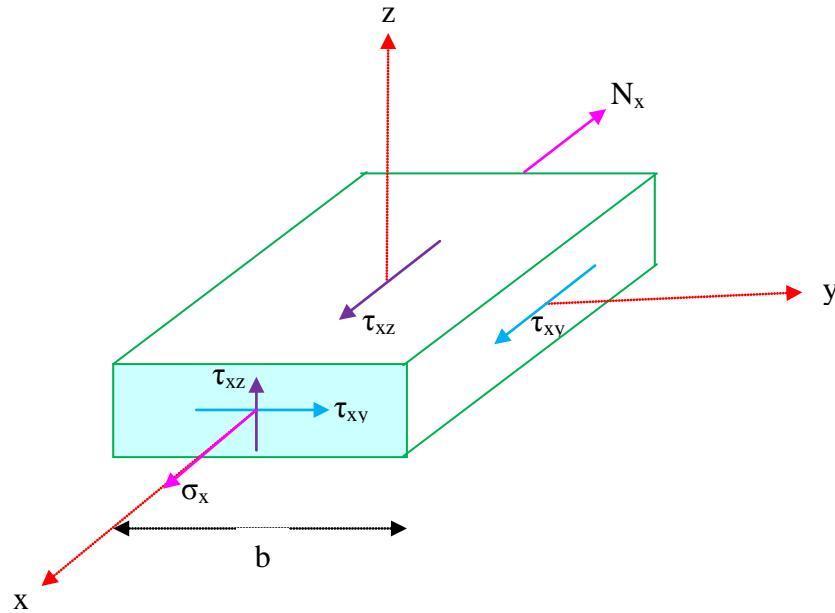


Figure 5.1a: Stresses in a laminate

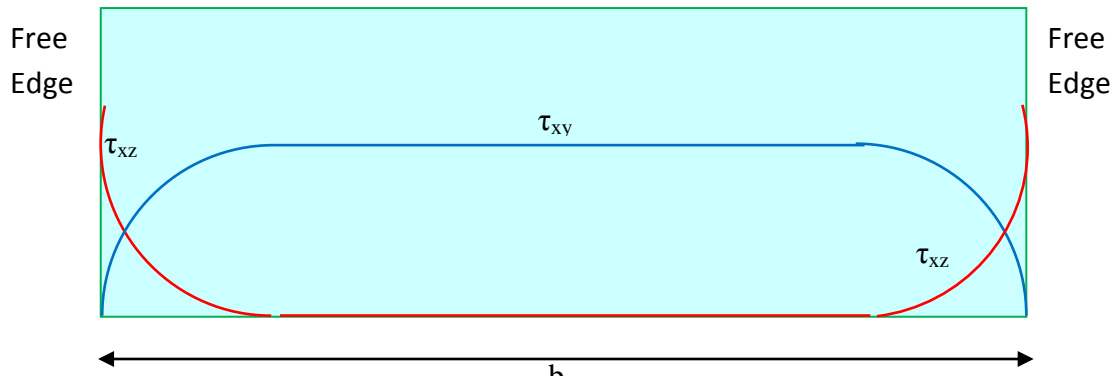


Figure 5.1b. Inter-laminar shear stresses

Suppose, when the layers of a laminate are kept together without any bond between them, each layer will deform freely and undergo different transverse strains ϵ_y due to their different Poisson's ratios on the application of an axial load in the x-direction. On the other hand, if the layers are in perfect bonding, all the layers should experience an identical transverse strain ϵ_y . But, due to the mismatch of Poisson's ratios of different layers, each layer will face some constraint

against its free transverse deformation. Owing to that constraint, normal stress σ_y is produced in each lamina and inter-laminar shear stress τ_{yz} at the lamina interfaces.

Similarly, in a perfectly bonded laminate each lamina is limited to have equal shear strain γ_{xy} . But, due to the differences in the coefficients of mutual influence m_x , the inter-laminar shear stress τ_{xz} is produced.

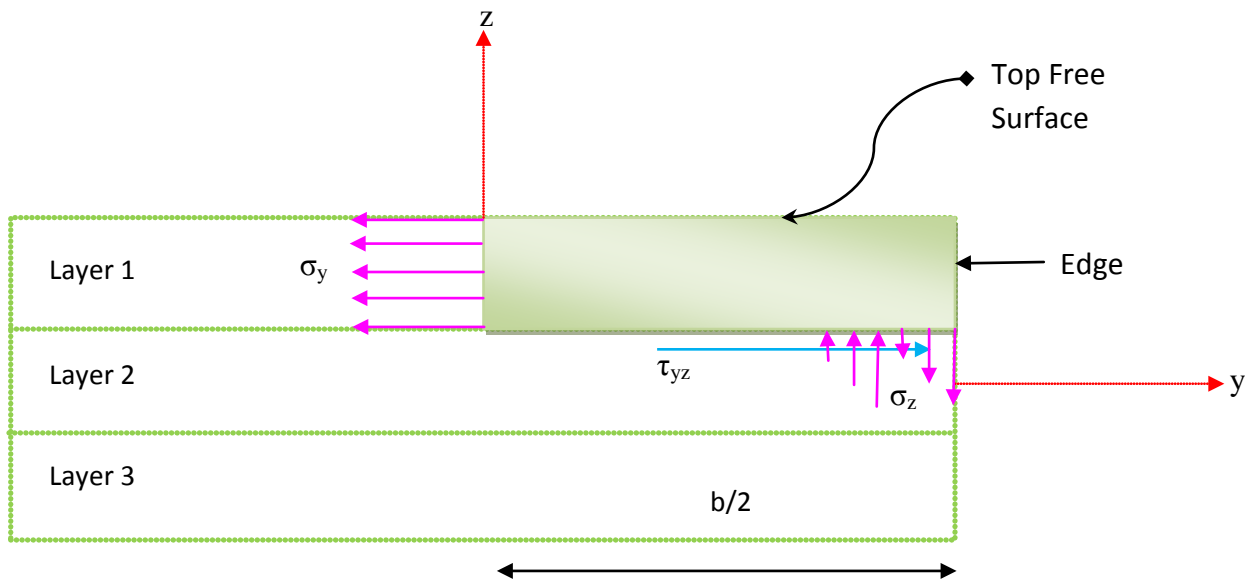


Figure 5.2 Inter-laminar stresses

From the figure 5.2, it is clear that along the y -direction the force equilibrium is maintained by σ_y and τ_{yz} . But, the force resultants of σ_y and τ_{yz} are not collinear. Due to non-collinear of the force resultants there will be some net moment to be balanced about the x -axis in the lamina. This net moment is balanced by the moment due to the inter-laminar normal stress σ_z . Thus, the moment equilibrium about the x -axis is satisfied.

Some important points to be remembered:

- (1) Inter-laminar stresses σ_z , τ_{xz} , and τ_{yz} are determined by numerical methods.
- (2) Inter-laminar stresses are due to the mismatch of Poissons' ratios and the m_x , between different layers, not due to the mismatch on elastic and shear moduli.
- (3) Inter-laminar stresses can be noticed significantly near the free edge. As a result, delamination may initiate at the free edges.
- (4) For an $[\theta / -\theta]$ angle-ply laminate in uniaxial tension,

τ_{xz} is the most significant stress than other inter-laminar stresses at $\theta/-\theta$ interface. Its magnitude and direction depend on the fiber orientation angle, θ . Moreover, the effect of τ_{xz} will be severe in clustered $[\theta_n/-\theta_n]_s$ laminates than alternating $[(\theta/-\theta)_n]_s$ laminates.

(5) For a $[0^\circ/90^\circ]$ cross-ply laminate in uniaxial tension,

σ_z and τ_{yz} are the most significant inter-laminar stresses. Their magnitude, direction and locations depend on stacking sequences of the laminate.

For example:

In $[0^\circ/90^\circ/90^\circ/0^\circ]$ laminate, σ_z is maximum at the mid plane of the laminate and the nature of σ_z is tensile. So, delamination is expected.

In $[90^\circ/0^\circ/0^\circ/90^\circ]$ laminate, σ_z is maximum at the mid plane but compressive in nature.

(6) for a general laminate,

all the inter-laminar stresses σ_z , τ_{xz} , and τ_{yz} are present.

For example:

In $[45^\circ/-45^\circ/0^\circ/0^\circ/-45^\circ/45^\circ]$ laminate under uniaxial tensile load,

τ_{xz} is higher in $[\theta/-\theta]$ laminate interface than in $[0^\circ/-45^\circ]$ laminate interface. τ_{yz} is higher in $[0^\circ/-45^\circ]$ laminate interface than in $[\theta/-\theta]$ laminate interface. σ_z is maximum at the mid plane of the laminate.

(7) stacking sequence has a strong influence on the nature, magnitude and location of inter-laminar stresses.

(8) Material properties also have a strong influence.

References:

"Mechanics of Composite Structural Elements", H Altenbach, J Altenbach and W Kissing, Springer publications.

" Principles of Composite Material Mechanics", Ronald F Gibson, CRC Press.

"Fiber-Reinforced Composites", P K Mallick, CRC Press.

Mechanics of Laminated Composite Plates, J N Reddy, CRC Publications

"Analysis and Performance of Fiber composites", Agarwal, Broughtman and Chandrashekhara, John Wiley and sons, Inc.